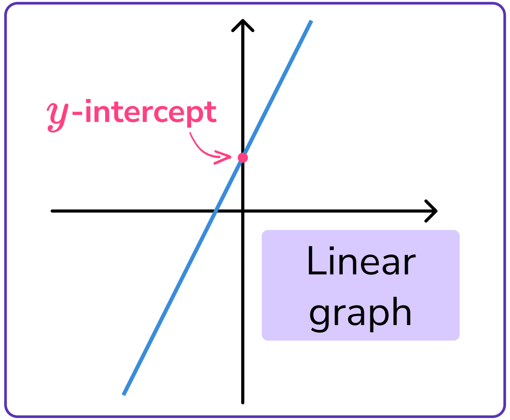
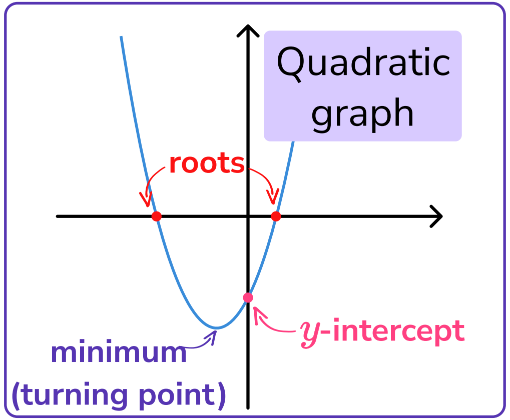
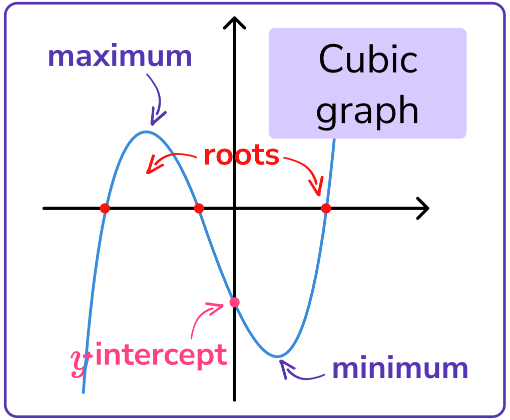
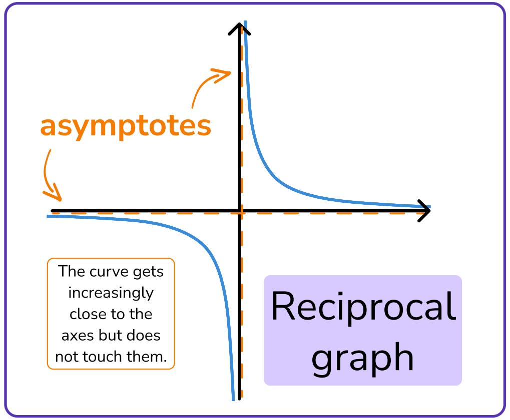
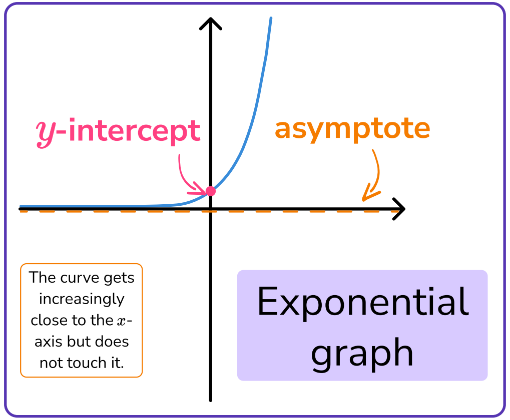
Types of Graphs

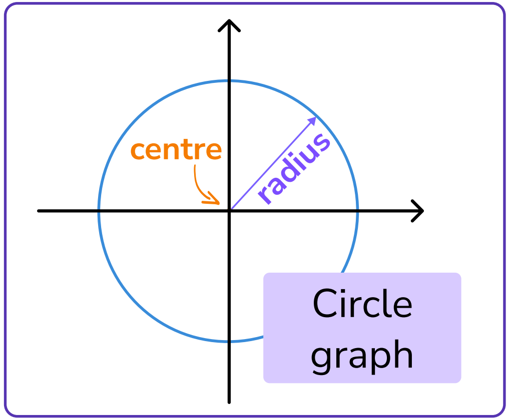












Straight Line Graphs

A **straight line graph** is a visual representation of a linear function.

A straight line has a general equation of

$$y = mx + c$$

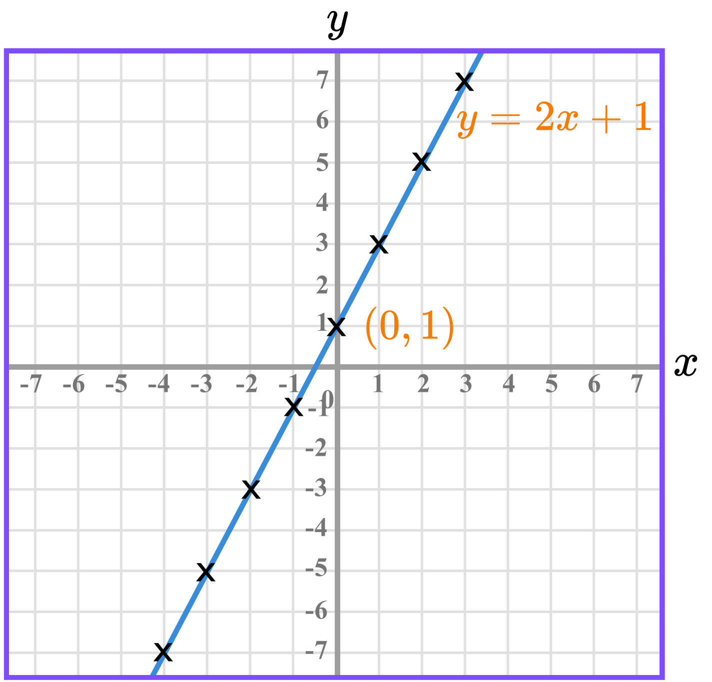
gradient   y-intercept

Example

$y = 2x + 1$

$m = 2$, and $c = 1$

The graph of this equation looks like this:



THIRD SPACE LEARNING

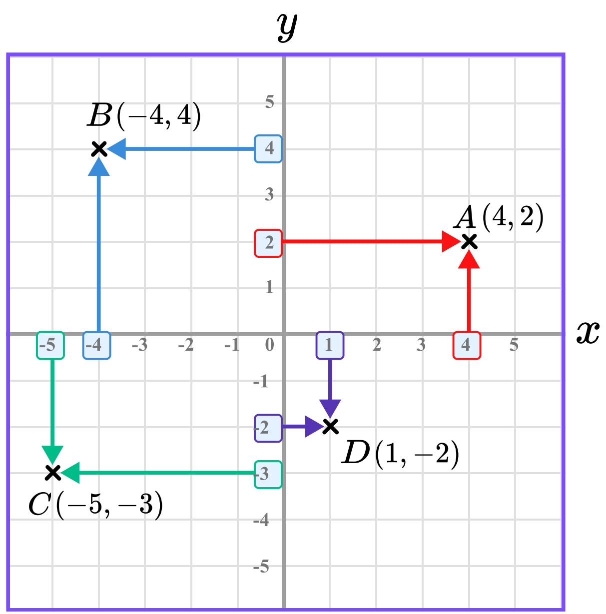
Coordinates

Coordinates are locations of points on a grid, known as the Cartesian plane.

A coordinate is written as two numbers, separated by a comma, and surrounded by a pair of round brackets.

The general form of a coordinate is (x, y)

To determine the location of a coordinate on the Cartesian plane, we need to know its horizontal and vertical location.



Gradient of a Line

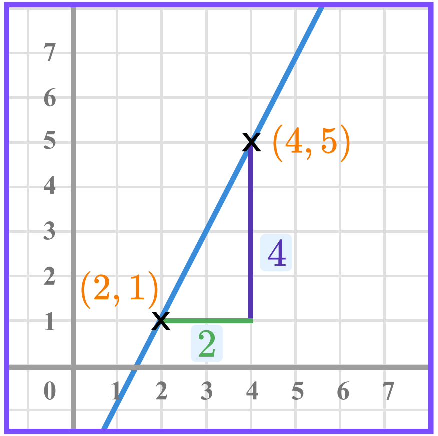
The **gradient of a line** shows how steep the straight line is. In the general equation of straight line, $y = mx + c$, the gradient is denoted by the letter m.

To calculate the gradient of a straight line through two coordinates (x_1, y_1) and (x_2, y_2):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example

$$m = \frac{5-1}{4-2} = \frac{4}{2} = 2$$



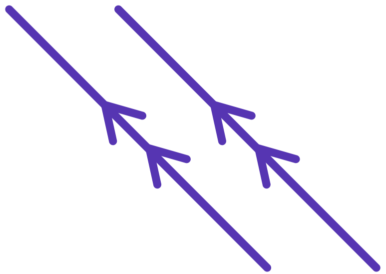
It can be helpful to think about this formula as: **"change in y divided by change in x"** or **"rise over run"**.

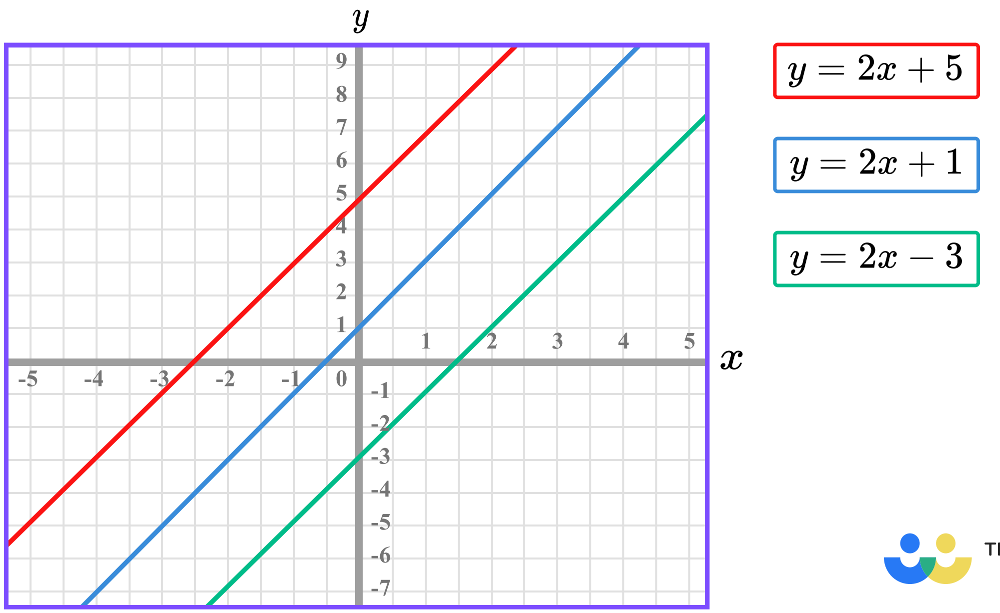
Parallel Lines

Parallel lines are straight lines with a constant distance between them.

They share the **same gradient.**

Example





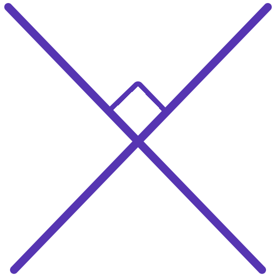
Perpendicular Lines

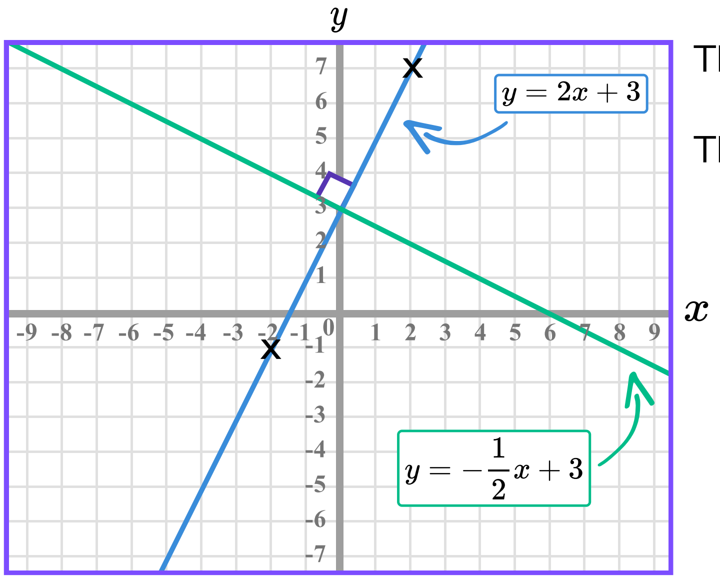
Perpendicular lines intersect (cross) one another at 90° (a right angle).

They have **gradients that multiply to give** -1

Negative reciprocal

Example





The line $y = 2x + 3$ has a gradient of 2

The line $y = -\frac{1}{2}x + 3$ has a gradient of $-\frac{1}{2}$

$$2 \times -\frac{1}{2} = \frac{-2}{2} = -1$$

The gradients multiply to give -1

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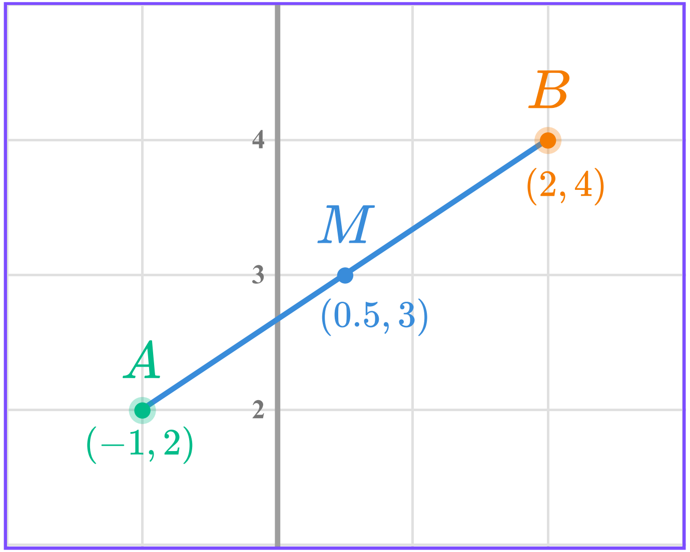
Midpoint of a Line

The **midpoint of a line** segment is a point that lies exactly halfway between two points.



Example

Average of the x coordinates is $\frac{-1+2}{2}=\frac{1}{2}=0.5$

Average of the y coordinates is $\frac{2+4}{2}=\frac{6}{2}=3$

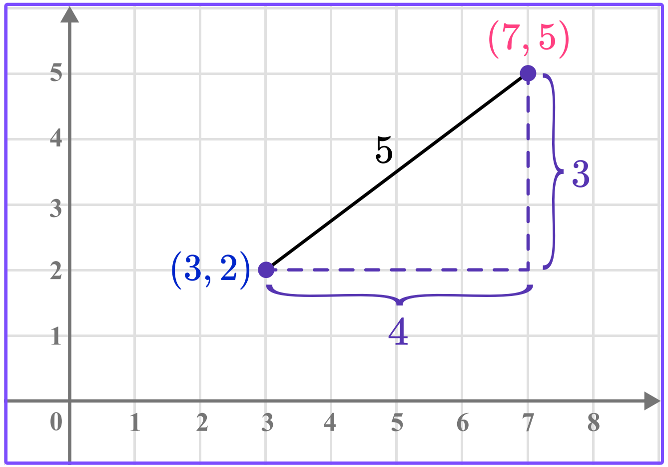
The midpoint of the line AB is $M=(0.5,3)$

Distance Formula

The **distance formula** calculates the distance d between two coordinates (x_1, y_1) and (x_2, y_2) on an xy-coordinate plane.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example Find the distance between the points $(3, 2)$ and $(7, 5)$



$$\begin{aligned} d &= \sqrt{(7 - 3)^2 + (5 - 2)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

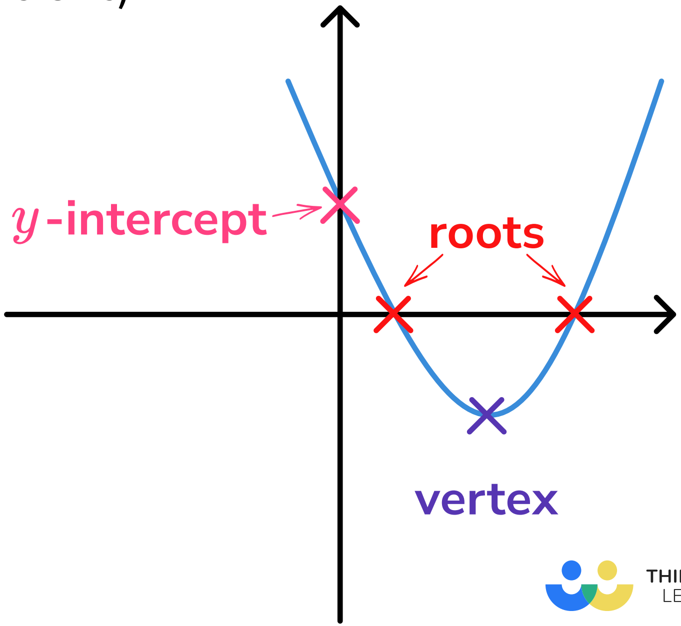
Use Pythagoras' Theorem

Quadratic Graphs

Quadratic graphs are graphs of quadratic functions and are u-shaped (positive x^2 coefficient) or n-shaped (negative x^2 coefficient).
The shape of the graph is called a parabola.

The key features are:

- The roots or solutions (where the graph touches or crosses the x-axis)
- The y-intercept
- The vertex (also called the turning point).

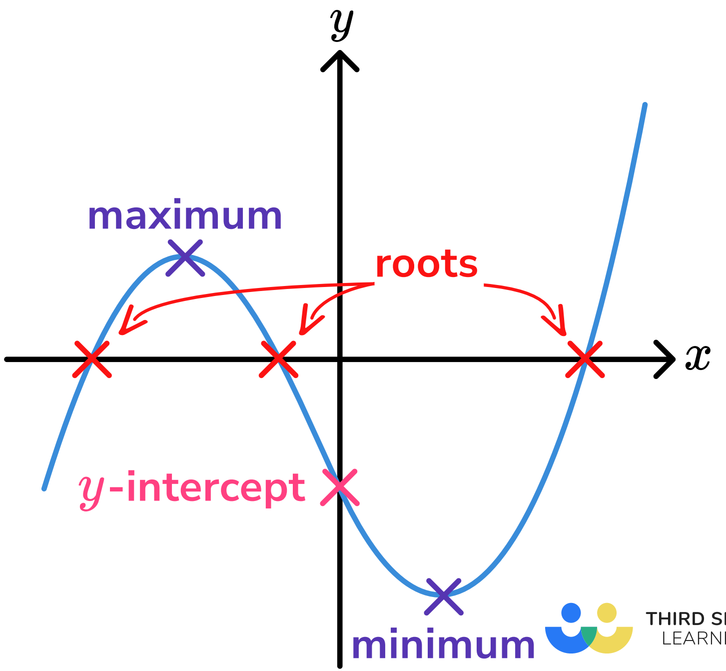


Cubic Graph

A **cubic graph** is a graphical representation of a cubic function.
A cubic is a polynomial which has an x^3 term as the highest power of x.

Some cubic graphs have two turning points - a **minimum point** and a **maximum point**.

A cubic graph with two turning points can touch or cross the x axis between one and three times.



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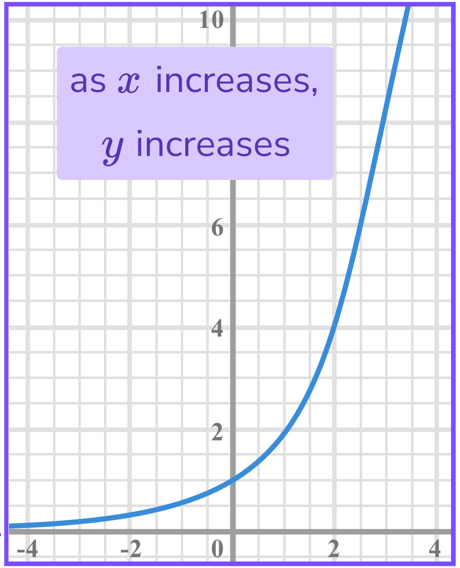
Exponential Graph

An **exponential graph** is a representation of an exponential function of the form: $y = k^x$

Examples

$y = 2^x$

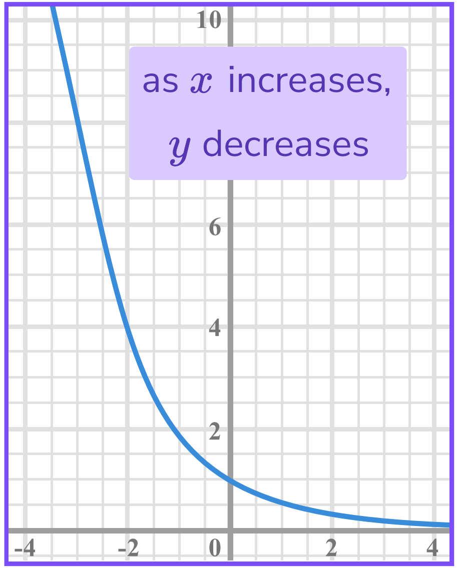
The graph will never touch the x-axis



Exponential growth
$k > 1$

In these examples the y-intercept (at $x = 0$) is 1 since anything raised to the power 0 is 1

$y = (\frac{1}{2})^x$



Exponential decay
$k < 1$

The graph will never touch the x-axis

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Reciprocal Graph

A common form of a reciprocal graph is:

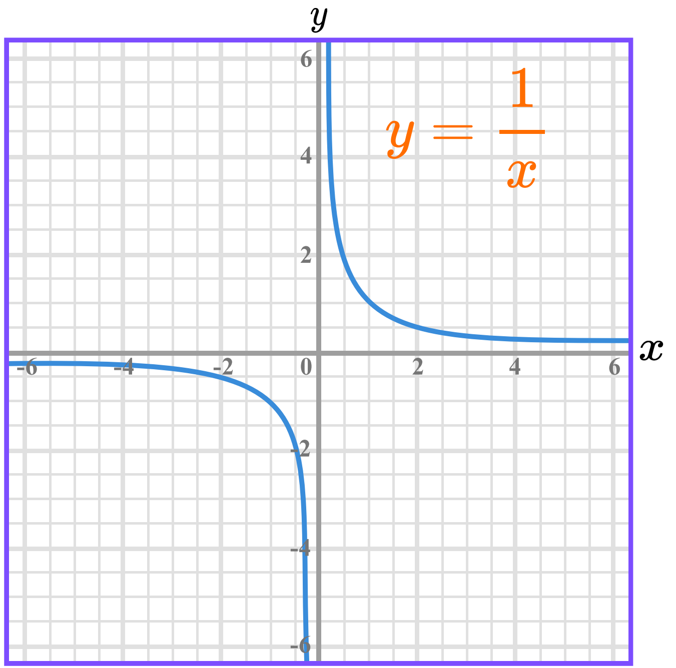
$$y=\frac{c}{x}$$

$\boldsymbol{c}$ is a constant

or $xy=c$

This form of reciprocal graph is a smooth curve called a hyperbola. It has two branches.

Example $y=\frac{1}{x}$



Here, the curve gets very close to the x and y axes but never touches them.

This means that the x and y axes are **asymptotes** to the curve.

Circle Graph

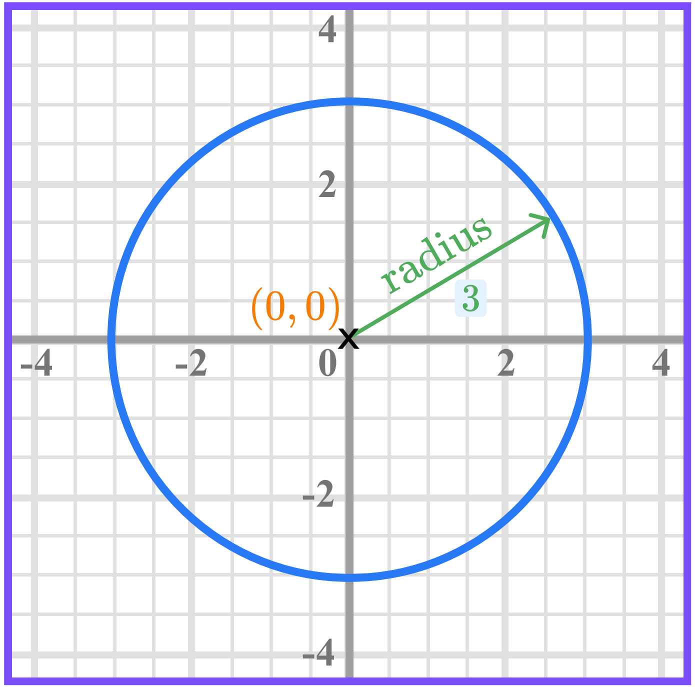
The general equation for a circle with centre $(0,0)$ and radius r is $x^2+y^2=r^2$

Example

The circle with equation $x^2+y^2=9$ has:

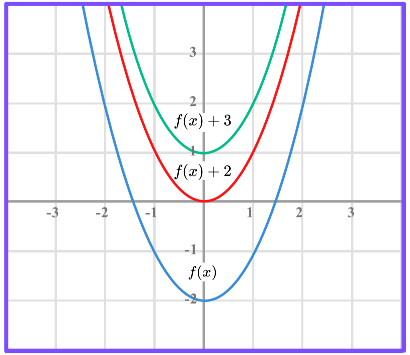
- radius 3
- centre $(0,0)$

$r^2=9$
$r=\pm\sqrt{9}$
$=\pm 3$
The radius of the circle is 3



Graph Transformations - Translations

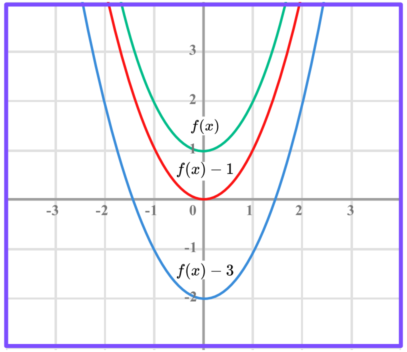
Vertical translations



$f(x)+a$

Translation by vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$ $\uparrow$

Add a to the y coordinate

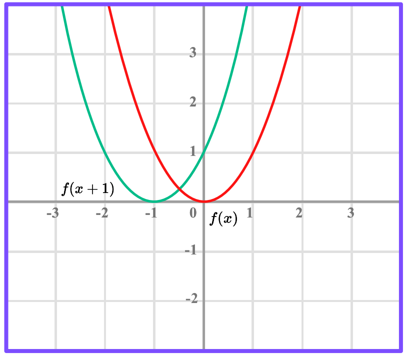


$f(x)-a$

Translation by vector $\begin{pmatrix} 0 \\ -a \end{pmatrix}$ $\downarrow$

Subtract a from the y coordinate

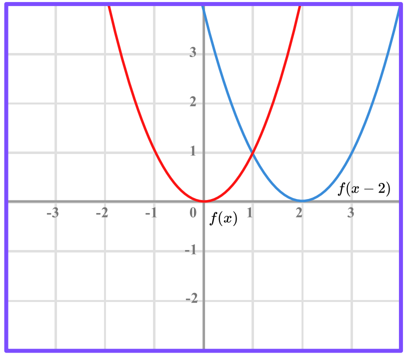
Horizontal translations



$f(x+a)$

Translation by vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$ $\leftarrow$

Subtract a from the x coordinate



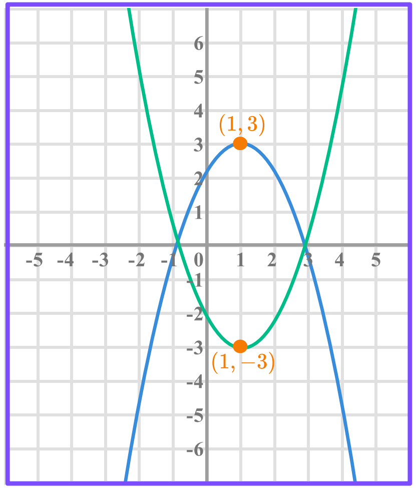
$f(x-a)$

Translation by vector $\begin{pmatrix} a \\ 0 \end{pmatrix}$ $\rightarrow$

Add a to the x coordinate

Graph Transformations - Reflections

Reflections in the x axis

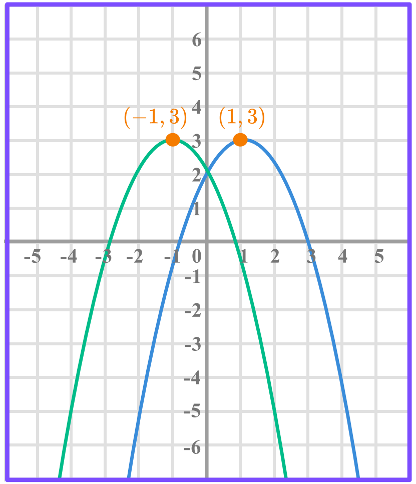


$-f(x)$

Reflection in the x-axis

Multiply the y coordinates by -1

Reflections in the y axis



$f(-x)$

Reflection in the y-axis

Multiply the x coordinates by -1